Indian Statistical Institute, Bangalore Centre
B.Math. (I Year) : 2011-2012
Semester I : Semestral Examination
Probability Theory I

30.11.2011 Time: 3 hours. Maximum Marks : 100

Note: The paper carries 105 marks. Any score above 100 will be taken as 100. State clearly the results you are using in your answers.

1. (10+10+10 = 30 marks) Suppose n distinct balls are distributed at random into 3 distinct boxes. Let X = number of balls in Box 1 and Y = number of balls in Box 2.

   (i) Find the discrete density function of the two-dimensional discrete random variable (X, Y).

   (ii) Find the marginal density functions.

   (iii) Let $0 \leq k \leq (n-1)$. Show that the conditional density of X given $Y = k$ is a binomial density function, indicating the parameters.

2. (12 marks) Let X, Y be independent discrete random variables having Poisson distribution with respective parameters $\lambda_1, \lambda_2 > 0$. Find the distribution of $X + Y$.

3. (8+8+8 = 24 marks) Let C be a constant. Define the function f by

$$\begin{aligned} f(x) &= Cx\exp(-x^2), \quad \text{if} \quad x > 0 \\ &= 0, \quad \text{otherwise.} \end{aligned}$$

   (i) Find C so that f is a probability density function.

   (ii) Find the corresponding distribution function.

   (iii) Let X be a real valued absolutely continuous random variable with f as its probability density function. Find the probability density function of X^2.

4. (12 marks) Let X be an absolutely continuous random variable having an exponential distribution with parameter $\lambda > 0$. Define the discrete random variable Y by $Y = k$ if $k \leq X < (k + 1)$ for $k = 0, 1, 2, \ldots$. Find the distribution of Y.

5. (12 marks) Let X be a random variable having standard normal distribution; let Φ denote its distribution function. Find

$$\int_0^\infty \text{Prob. } (\Phi(X) \geq u)du.$$

6. (15 marks) Let X be a random variable having $N(\mu, \sigma^2)$ distribution. Find the moment generating function of X.